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POWER SERIES FOR THE CALCULATION OF BESSEL FUNCTIONS FOR LARGE --ETC(U)

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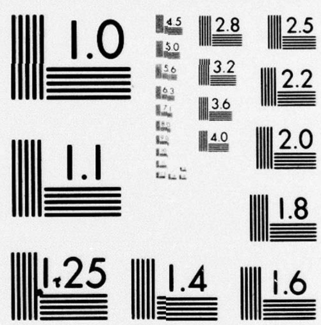

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POWER SERIES FOR THE CALCULATION OF BESSEL FUNCTIONS FOR LARGE ARGUMENTS

Provides $J_\nu(x)$ and $Y_\nu(x)$, accurate to at least 14 places.

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NAVAL OCEAN SYSTEMS CENTER
SAN DIEGO, CALIFORNIA 92152

NAVAL OCEAN SYSTEMS CENTER, SAN DIEGO, CA 92152

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SL GUILLE, CAPT, USN
Commander

HL BLOOD
Technical Director

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In certain applications such as acoustic wave scattering from internal waves, the Bessel functions $J_\nu(x)$ and $Y_\nu(x)$ for real order and large argument are required to be known accurately to at least 14 figures. Higher order terms in the power series developed by Goldstein and Thaler are presented herein. Applying these results allows $J_\nu(x)$ and $Y_\nu(x)$ to be determined accurately to at least 14 figures.

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INTRODUCTION

Goldstein and Thaler (ref 1) have employed the phase-amplitude method to develop mathematical expressions for the calculation of Bessel functions of real order and argument for large values of the argument. The power series reported in reference 1 allow $J_\nu(x)$ and $Y_\nu(x)$ to be calculated within an accuracy of six to eight figures. Presented herein are higher order terms of the power series that allow $J_\nu(x)$ and $Y_\nu(x)$ to be calculated within an accuracy of 14 to 16 figures.

THE POWER SERIES

It can be shown (reference 1) that the Bessel functions $J_\nu(x)$ and $Y_\nu(x)$ can be written in terms of an amplitude and a phase function:

$$J_\nu(x) = \left[\frac{B_\nu(t)}{t}\right] \sqrt{\frac{2t}{\pi}} \cos \Phi_\nu(t)$$

and

$$Y_\nu(x) = \left[\frac{B_\nu(t)}{t}\right] \sqrt{\frac{2t}{\pi}} \sin \Phi_\nu(t) \ .$$

The amplitude function is expanded into a power series about $t = 0$:

$$[B_\nu(t)/t] = 1 + a_2 t^2 + a_4 t^4 + a_6 t^6 + a_8 t^8 + a_{10} t^{10} + a_{12} t^{12} + a_{14} t^{14} + \ldots ,$$

where

$$a_2 = \frac{1}{4}\alpha$$

$$a_4 = \frac{5}{32}\alpha^2 - \frac{3}{8}\alpha$$

$$a_6 = \frac{15}{128}\alpha^3 - \frac{37}{32}\alpha^2 + \frac{15}{8}\alpha$$

$$a_8 = \frac{195}{2048}\alpha^4 - \frac{611}{256}\alpha^3 + \frac{1821}{128}\alpha^2 - \frac{315}{16}\alpha$$

$$a_{10} = \frac{663}{8192}\alpha^5 - \frac{4199}{1024}\alpha^4 + \frac{29\,811}{512}\alpha^3 - \frac{2223}{8}\alpha^2 + \frac{2835}{8}\alpha$$

$$a_{12} = \frac{4641}{65\,536}\alpha^6 - \frac{103\,649}{16\,384}\alpha^5 + \frac{713\,065}{4096}\alpha^4 - \frac{1\,944\,363}{1024}\alpha^3 + \frac{513\,495}{64}\alpha^2 - \frac{155\,925}{16}\alpha$$

1. Bessel Functions for Large Arguments, by M Goldstein and RM Thaler; Mathematical Tables and Other Aids to Computation, vol 13, no 65, 1958, p 18-26

$$a_{14} = \frac{16\,575}{262\,144}\alpha^7 - \frac{595\,595}{65\,536}\alpha^6 + \frac{438\,325}{1024}\alpha^5 - \frac{35\,994\,327}{4096}\alpha^4 + \frac{84\,480\,939}{1024}\alpha^3$$

$$- \frac{41\,365\,215}{128}\alpha^2 + \frac{6\,081\,075}{16}\alpha$$

The phase function is similarly expanded into a power series about $t = 0$:

$$t\left[\Phi_\nu + \left(\nu + \frac{1}{2}\right)\frac{t}{2}\right] = 1 + b_2 t^2 + b_4 t^4 + b_6 t^6 + b_8 t^8 + b_{10} t^{10} + b_{12} t^{12}$$

$$+ b_{14} t^{14} + \ldots ,$$

where

$$b_2 = \frac{1}{2}\alpha$$

$$b_4 = \frac{1}{24}\alpha^2 - \frac{\alpha}{4}$$

$$b_6 = \frac{1}{80}\alpha^3 - \frac{7}{20}\alpha^2 + \frac{3}{4}\alpha$$

$$b_8 = \frac{10}{1792}\alpha^4 - \frac{95}{224}\alpha^3 + \frac{807}{224}\alpha^2 - \frac{315}{56}\alpha$$

$$b_{10} = \frac{7}{2304}\alpha^5 - \frac{35}{72}\alpha^4 + \frac{1975}{192}\alpha^3 - 58\alpha^2 + \frac{315}{4}\alpha$$

$$b_{12} = \frac{21}{11\,264}\alpha^6 - \frac{3045}{5632}\alpha^5 + \frac{64\,319}{2816}\alpha^4 - \frac{427\,071}{1408}\alpha^3 + \frac{247\,365}{176}\alpha^2 - \frac{155\,925}{88}\alpha$$

$$b_{14} = \frac{37\,751}{30\,457\,856}\alpha^7 - \frac{3927}{6656}\alpha^6 + \frac{145\,013}{3328}\alpha^5 - \frac{1\,851\,351}{1664}\alpha^4 + \frac{19\,472\,013}{1664}\alpha^3$$

$$- \frac{10\,091\,115}{208}\alpha^2 + \frac{6\,081\,075}{104}\alpha$$

and

$$\alpha \equiv \left(\nu^2 - \frac{1}{4}\right) .$$

Attention is also called to a typographical error in reference 1 for b_8. The form shown herein is the correct one.

CONCLUSION

The power series developed by Goldstein and Thaler are extended to allow the Bessel functions $J_\nu(x)$ and $Y_\nu(x)$ to be calculated accurately to at least 14 figures.

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